- $$\lim_{x \to 0} \frac{\arctan(2x) - 2\sqrt{1+2x} + 2}{\ln(1+5x) - 5\sin x}$$

Denom

$$D(x) = \ln(1+5x) - 5\sin x$$

$$\ln(1+5x) \overset{y=5x}{=} \ln(1+y)$$
$$= y - \frac{y^2}{2} + o(y^2)$$
$$= 5x - \frac{1}{2}(5x)^2 + o((5x)^2)$$
$$= 5x - \frac{25}{2}x^2 + o(x^2)$$

$$\sin x = x - \frac{1}{3!}x^3 + o(x^3)$$
$$= x - \frac{1}{6}x^3 + o(x^3)$$

$$D(x) = \ln(1+5x) - 5\sin x$$
$$= 5x - \frac{25}{2}x^2 + o(x^2) - 5\left(x - \frac{1}{6}x^3 + o(x^3)\right)$$
$$= \cancel{5x} - \frac{25}{2}x^2 + o(x^2) - \cancel{5x} + \frac{5}{6}x^3 + o(x^3)$$
$$= -\frac{25}{2}x^2 + o(x^2)$$

Num

$$N(x) = \arctan(2x) - 2\sqrt{1+2x} + 2$$

$$\arctan(2x) \overset{y=2x}{=} \arctan y$$
$$= y - \frac{y^3}{3} + o(y^3)$$
$$= 2x - \frac{1}{3}(2x)^3 + o((2x)^3)$$
$$= 2x - \frac{8}{3}x^3 + o(x^3)$$

$$\sqrt{1+2x} \overset{y=2x}{=} \sqrt{1+y}$$
$$= (1+y)^{\frac{1}{2}} \quad {=\alpha}$$
$$= 1 + \frac{1}{2}y + \frac{\overbrace{\frac{1}{2}\left(\frac{1}{2}-1\right)}^{-\frac{1}{8}}}{2}y^2 + o(y^2)$$

$$(1+y)^\alpha = 1 + \alpha y + \frac{\alpha(\alpha-1)}{2}y^2 + o(y^2)$$

$$= 1 + \frac{1}{\cancel{2}} \cdot \cancel{2}x - \frac{1}{8} \cdot (2x)^2 + o(x^2)$$

$$= 1 + x - \frac{1}{\underset{2}{\cancel{8}}} \cdot \cancel{4}x^2 + o(x^2)$$

$$-2\sqrt{1+2x} = -2\left(1 + x - \frac{1}{2}x^2 + o(x^2)\right)$$

$$= -2 - 2x + x^2 + o(x^2)$$

$$N(x) = \cancel{2x} - \frac{8}{3}x^3 + o(x^3) - \cancel{2} - \cancel{2x} + x^2 + o(x^2) \cancel{+2}$$

$$= x^2 + o(x^2)$$

$$\lim_{x \to 0} \frac{x^2 + o(x^2)}{-\frac{25}{2}x^2 + o(x^2)} = \frac{1}{-\frac{25}{2}} = -\frac{2}{25}$$

- $$\lim_{x \to 0} \frac{(e^{2x} - 1)^2 - 4x \sin x}{x \cos x - \sin x}$$

$$D(x) = x \cos x - \sin x$$

$$x \cos x = x\left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + o(x^4)\right)$$

$$= x - \frac{1}{2}x^3 + \frac{1}{24}x^5 + o(x^5)$$

$$\sin x = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + o(x^5)$$

$$D(x) = \cancel{x} - \frac{1}{2}x^3 + \frac{1}{24}x^5 + o(x^5) - \cancel{x} + \frac{1}{6}x^3 - \frac{1}{120}x^5 + o(x^5)$$

$$= \left(-\frac{1}{2} + \frac{1}{6}\right)x^3 + \underbrace{\left(\frac{1}{24} - \frac{1}{120}\right)x^5 + o(x^5)}_{} = o(x^3)$$

$$= -\frac{1}{3}x^3 + o(x^3)$$

$$N(x) = (e^{2x} - 1)^2 - 4x \sin x$$

$$e^{2x} \overset{y=2x}{=} e^y$$

$$= 1 + y + \frac{1}{2}y^2 + o(y^2)$$

$$= 1 + 2x + \frac{1}{2}(2x)^2 + o((2x)^2)$$

$$= 1 + 2x + \frac{1}{2}4x^2 + o(x^2)$$

$$= 1 + 2x + 2x^2 + o(x^2)$$

$$(e^{2x}-1)^2 = (1 + 2x + 2x^2 + o(x^2) - 1)^2$$
$$= (2x + 2x^2 + o(x^2))^2$$
$$= 4x^2 + 4x^4 + o(x^4) + \underbrace{8x^3}_{2\cdot 2x \cdot 2x^2} + o(x^3) + o(x^4)$$
$$= 4x^2 + 8x^3 + o(x^3)\;!\,.$$

$$-4x \sin x = -4x\left(x - \frac{x^3}{6} + o(x^3)\right)$$
$$= -4x^2 + \frac{2}{3}x^4 + o(x^4)\,.$$

$$N(x) = \cancel{4x^2} + 8x^3 + o(x^3) - \cancel{4x^2} + \frac{2}{3}x^4 + o(x^4)$$
$$= 8x^3 + o(x^3)$$
$$\lim_{x\to 0}\frac{N(x)}{D(x)} = \lim_{x\to 0}\frac{8x^3+o(x^3)}{-\frac{1}{3}x^3+o(x^3)} = 8\cdot(-3) = -24$$

$$(e^{2x}-1)^2 = (e^{2x})^2 + 1 - 2e^{2x}$$
$$= e^{4x} + 1 - 2e^{2x}$$
$$= 1 + 4x + \frac{1}{2}(4x)^2 + \overbrace{o(x^2)}^{\frac{1}{6}(4x)^3+o(x^3)} + 1 - 2\Big(1 + 2x + \frac{1}{2}(2x)^2 + \overbrace{o(x^2)}^{+\frac{1}{6}(2x)^3+o(x^3)}\Big)$$
$$= \cancel{1} + \cancel{4x} + 8x^2 + o(x^2) + \cancel{1} - \cancel{2} - \cancel{4x} - 4x^2 + o(x^2)$$
$$= 4x^2 + \underbrace{o(x^2)}_{\substack{\frac{32}{3}x^3+o(x^3) - \frac{8}{3}x^3+o(x^3)\\ 8x^3+o(x^3)}}$$

Numeri complessi.

Introduciamo un numero i (UNITÀ IMMAGINARIA) che soddisfa $i^2=-1$.

Numeri complessi: $z = x + iy$ con $x, y \in \mathbb{R}$.

ESEMPI

$z = 2 + 3i$ $(x=2,\ y=3)$

$z = 3 - i$ $(x=3,\ y=-1)$

$z = \pi + 2i$ $(x=\pi,\ y=2)$

x è detto PARTE REALE

y è detto PARTE IMMAGINARIA

La scrittura

$z = \pi + 2i \qquad (x = \pi,\ y = 2)$

$z = 3i \qquad (x = 0,\ y = 3)$

$z = i \qquad (x = 0,\ y = 1)$

$z = 15 \qquad (x = 15,\ y = 0)$

La scrittura
$z = x + iy$ con $x, y \in \mathbb{R}$
è detta FORMA / RAPPRESENTAZIONE ALGEBRICA DI z.

$$\begin{aligned} z &= (2+3i)(5+i) \\ &= 10 + 2i + 15i + \underbrace{3i^2}_{3\cdot(-1) = -3} \\ &= 10 + 2i + 15i - 3 \\ &= \underbrace{7}_{\substack{\text{Parte reale di } z \\ x = 7}} + \underbrace{17i}_{\text{parte immaginaria } y = 17} \end{aligned}$$

A cosa servono i numeri complessi.
Con i numeri complessi si possono risolvere tutte le equazioni di secondo grado:

$x^2 + 3x + 5 = 0$

- Con i numeri reali:
  $\Delta = 9 - 4 \cdot 5 = -11 < 0$
  Non ci sono soluzioni reali.
- Con i numeri complessi;
  $\Delta = -11$
  $(\sqrt{11}\, i)^2 = 11\, i^2 = -11 = \Delta$ quindi $\pm\sqrt{-11} = \pm\sqrt{11}\, i$
  le soluzioni complesse $z^2 + 3z + 5 = 0$ sono
  $z_{1,2} = \dfrac{-3 \pm \sqrt{11}\, i}{2} = -\dfrac{3}{2} \pm \dfrac{\sqrt{11}}{2} i$.

$z^2 + 4 = 0$

$z^2 = -4$

$z^2 = i^2 \cdot 4$

$z^2 = (2i)^2$

$z = \pm 2i$

Se $a \in \mathbb{R}$, $a < 0$

Se $a \in \mathbb{R}$, $a < 0$

$z^2 = a \Rightarrow z = \pm i \sqrt{|a|}$

L'insieme dei numeri complessi si indica con $\mathbb{C}$.

$\mathbb{C} = \{ x + iy \mid x, y \in \mathbb{R} \}$.

ESERCIZIO

Determinare la forma algebrica del numero complesso

$z = \dfrac{3+i}{1+i}$ $\qquad \left(\dfrac{1}{2+\sqrt{3}} = \dfrac{1}{2+\sqrt{3}} \cdot \dfrac{2-\sqrt{3}}{2-\sqrt{3}} = \dfrac{2-\sqrt{3}}{4-(\sqrt{3})^2} \right)$

moltiplicare e dividere per $1-i$ $\qquad \left(\dfrac{1}{1+i} = \dfrac{1}{1+\sqrt{-1}} \right)$

$$z = \frac{3+i}{1+i} = \frac{(3+i)(1-i)}{(1+i)(1-i)} = \frac{3-3i+i-i^2}{1^2-i^2}$$

$$= \frac{3-2i+1}{1+1} = \frac{4-2i}{2}$$

$$= 2-i$$

$$z = \frac{6-7i}{1-2i} = \frac{(6-7i)(1+2i)}{(1-2i)(1+2i)}$$

$$= \frac{6+12i-7i-14i^2}{1-(2i)^2}$$

$$= \frac{6+5i+14}{1+4} = \frac{20+5i}{5} = 4+i.$$

$$z = \frac{2+3i}{3-2i}$$

$$= \frac{(2+3i)(3+2i)}{(3-2i)(3+2i)} = \frac{6+4i+9i+\overset{-6}{6i^2}}{9-(2i)^2} = \frac{13i}{9+4} = \frac{13i}{13} = i$$

$$z = \frac{3i-1}{}$$

$$z = \frac{3i - 1}{1 + i}$$

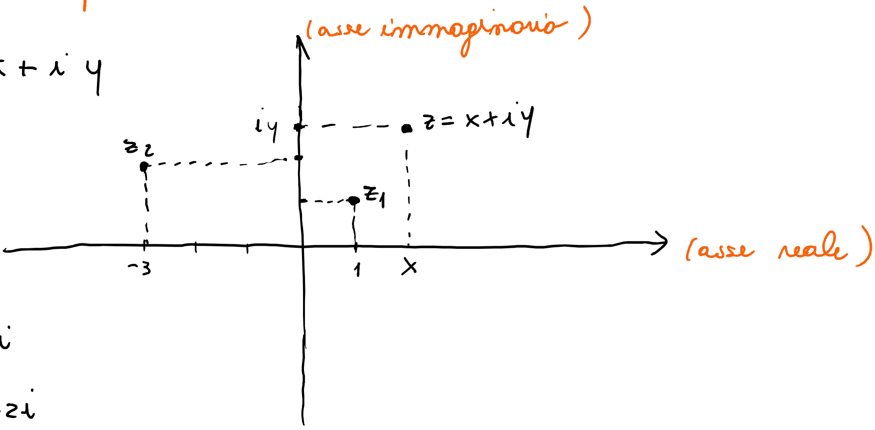
$$= \frac{3i - 1}{1 + i} \cdot \frac{1 - i}{1 - i} = \frac{3i - 3i^2 - 1 + i}{1 - i^2}$$

$$= \frac{3i + 3 - 1 + i}{1 + 1} = \frac{2 + 4i}{2} = 1 + 2i$$

Rappresentazione grafica dei numeri complessi

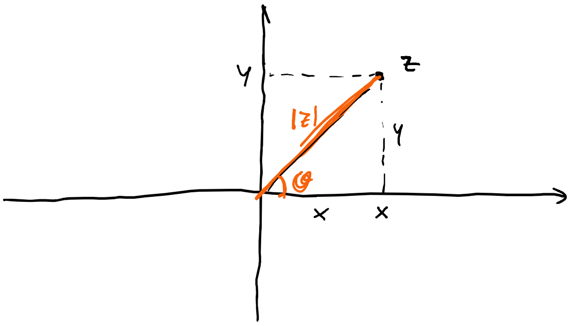
Piano complesso

$z = x + iy$



$z_1 = 1 + i$

$z_2 = -3 + 2i$



Se $z = x + iy$ la distanza da 0 è uguale a $\sqrt{x^2+y^2}$. Questa lunghezza è detta MODULO DI Z. e si indica con $|z|$ (o con $\|z\|$)

Relazioni tra x, y, $|z|$ e θ:

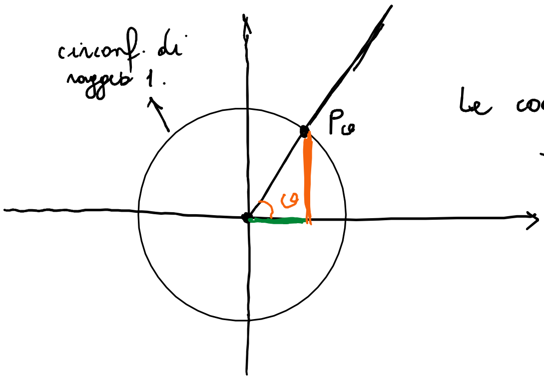
$$x = |z| \cos\theta \quad \text{e} \quad y = |z| \sin\theta.$$

- Se conosciamo x e y, allora:

  $|z| = \sqrt{x^2+y^2}$

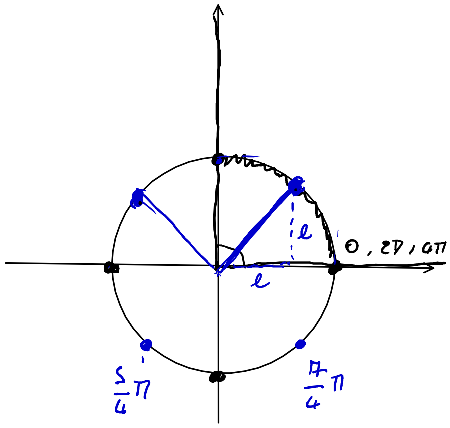
  θ è l'unico angolo t.c. $\cos\theta = \dfrac{x}{|z|}$ e $\sin\theta = \dfrac{y}{|z|}$

Valori noti di coseno e seno.

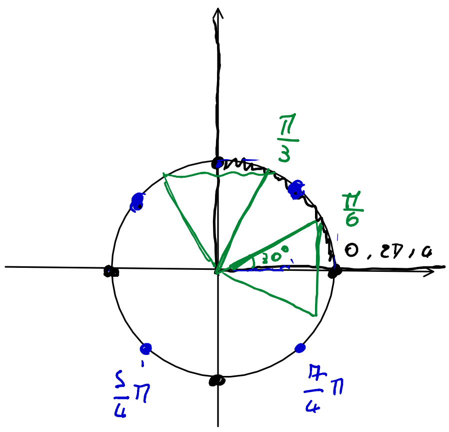


Le coordinate di P_ω sono il COSENO e il SENO di ω

gradi	radianti	$\cos\omega$	$\sin\omega$
0	0	1	0
90	$\frac{\pi}{2}$	0	1
180	π	-1	0
270	$\frac{3}{2}\pi$	0	-1
360	2π	1	0
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
135°	$\frac{3}{4}\pi$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
225°	$\frac{5}{4}\pi$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
315°	$\frac{7}{4}\pi$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
30°	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
60°	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$



$$\ell^2 + \ell^2 = 1$$
$$2\ell^2 = 1$$
$$\ell^2 = \frac{1}{2}$$
$$\ell = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$



$z = 1 + i$

Determiniamo $|z|$ e ω (ω è detto ARGOMENTO di z)

$z = 1 + i$

$x = 1, \quad y = 1$

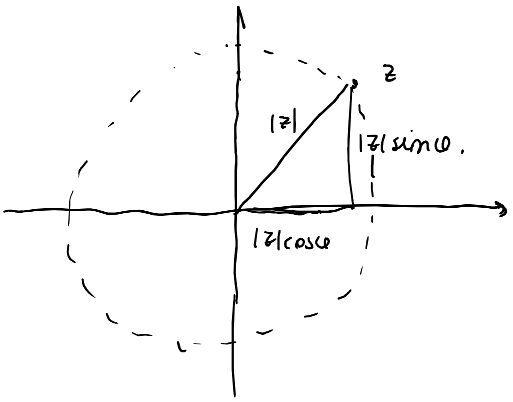
$|z| = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

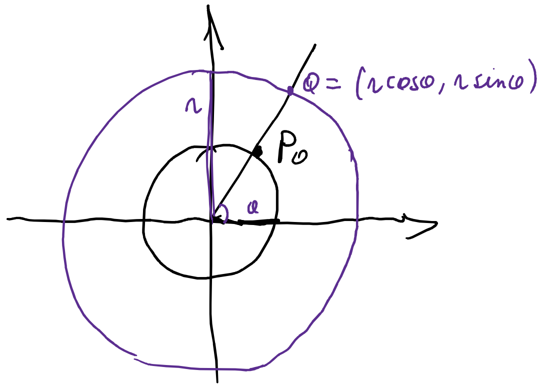
Come si trova ω? Abbiamo detto che è "l'unico" angolo

Come si trova θ? Abbiamo detto che è "l'unico" angolo t.c.

$$\cos\theta = \frac{x}{|z|} = \frac{1}{\sqrt{2}} \qquad e \qquad \sin\theta = \frac{y}{|z|} = \frac{1}{\sqrt{2}}$$

$$\Longrightarrow \quad \theta = \frac{\pi}{4}.$$





Esponenziali complessi

Dato $\theta \in \mathbb{R}$, definiamo

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Dato

$$\begin{aligned} z &= x + iy \\ &= |z|\cos\theta + i|z|\sin\theta \\ &= |z|(\cos\theta + i\sin\theta) \\ &= |z|e^{i\theta}. \end{aligned}$$

(FORMA ESPONENZIALE DI z)

ESEMPIO

$z = 1 + i$

$|z| = \sqrt{2}$ e $\theta = \frac{\pi}{4}$

Quindi la forma esponenziale di z è $z = \sqrt{2}\, e^{i\frac{\pi}{4}}$

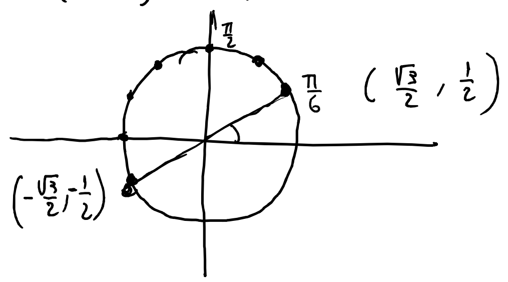
Si può dimostrare che

$$\left(|z|e^{i\theta}\right)^n = |z|^n \left(e^{i\theta}\right)^n = |z|^n e^{in\theta}$$
$$= |z|^n \left(\cos(n\theta) + i\sin(n\theta)\right)$$

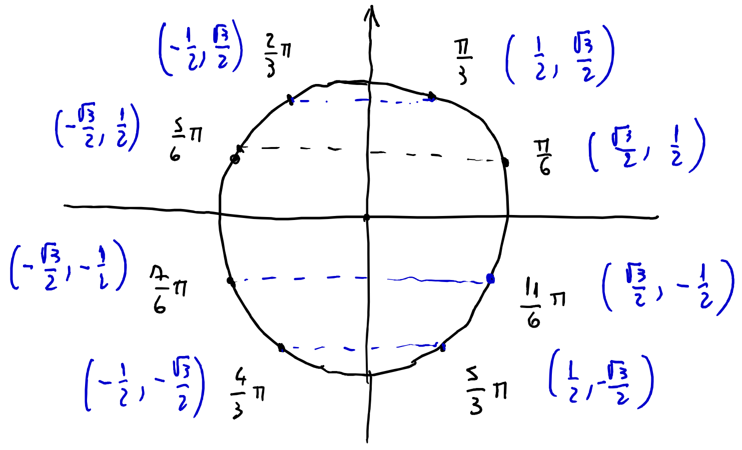
$$(1+i)^5 = \left(\sqrt{2}\, e^{i\frac{\pi}{4}}\right)^5 = (\sqrt{2})^5 e^{i\frac{5}{4}\pi}$$
$$= 4\sqrt{2}\left(\cos\left(\tfrac{5}{4}\pi\right) + i\sin\left(\tfrac{5}{4}\pi\right)\right)$$

$$= 4\sqrt{2}\left(\cos\left(\frac{5}{4}\pi\right) + i\sin\left(\frac{5}{4}\pi\right)\right)$$
$$= 4\sqrt{2}\left(-\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)$$
$$= -4 - 4i$$

$$\cos\left(\frac{7}{6}\pi\right) = ?$$

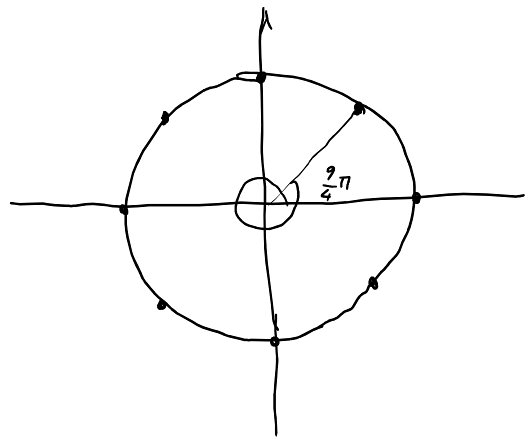


$$\cos\left(\frac{7}{6}\pi\right) = -\frac{\sqrt{3}}{2}$$



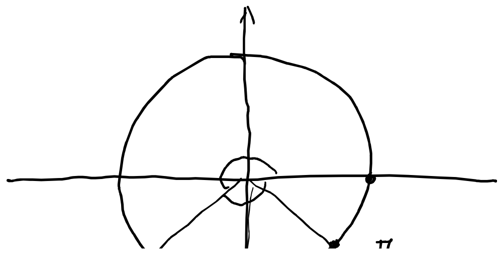
- $\cos\left(\frac{9}{4}\pi\right) = ?$

$$\cos\left(\frac{9}{4}\pi\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

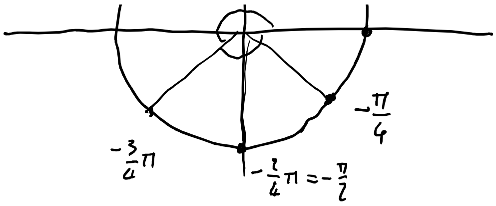


- $\cos\left(-\frac{3}{4}\pi\right)$

$$\cos\left(-\frac{3}{4}\pi\right) = \cos\left(\frac{5}{4}\pi\right)$$



$$\cos\left(-\frac{3}{4}\pi\right) = \cos\left(\frac{5}{4}\pi\right)$$
$$= -\frac{1}{\sqrt{2}}$$



FORMULA PER LE RADICI n-ESIME.

Sia $z \in \mathbb{C}$, $z = |z| e^{i\varphi}$ un numero complesso in forma esponenziale. Allora esistono n radici n-esime di z che sono i numeri del tipo:

$$w = \sqrt[n]{|z|}\, e^{i\left(\frac{\varphi}{n} + \frac{2k\pi}{n}\right)} \qquad \text{con } k = 0, 1, 2, \dots, n-1.$$

Formula per le potenze:

$$z^n = |z|^n e^{in\varphi}.$$

Formula per le radici n-esime:

$$\sqrt[n]{|z|}\, e^{i\left(\frac{\varphi}{n} + \frac{2k\pi}{n}\right)} \qquad \text{con } k = 0, 1, \dots, n-1$$

ESERCIZIO:

Sia $z = \dfrac{6+7i}{1+2i}$

1) Scrivere z in forma algebrica
2) Calcolare $(z-3)^9$

---

1) $$z = \frac{6+7i}{1+2i} \cdot \frac{1-2i}{1-2i} = \frac{6-12i+7i-14i^2}{1-(2i)^2} = \frac{6-12i+7i+14}{1+4}$$
$$= \frac{20-5i}{5} = 4-i.$$

2) Calcolare $(z-3)^9$ $\qquad |z-3| = \sqrt{x^2+y^2}$

$$z-3 = 4-i-3 = 1-i \qquad x=1,\ y=-1$$

Scriviamo questo numero in forma esponenziale.

$$|z-3| = \sqrt{1^2+(-1)^2} = \sqrt{2}$$

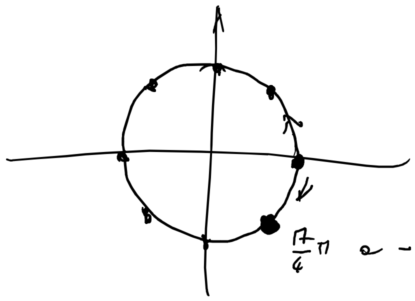
A

$$|z-3| = \sqrt{1^2+(-1)^2} = \sqrt{2}$$

Argomento (θ)

$$\cos\theta = \frac{1}{\sqrt{2}} \quad , \quad \operatorname{sen}\theta = -\frac{1}{\sqrt{2}}$$

$$\theta = \frac{7}{4}\pi \quad \text{oppure} \quad \theta = -\frac{\pi}{4}$$



$$(z-3)^9 = \left(\sqrt{2}\, e^{i\left(-\frac{\pi}{4}\right)}\right)^9$$

$$= (\sqrt{2})^9\, e^{i\left(-\frac{9}{4}\pi\right)}$$

$$(\sqrt{2})^8 = \left(2^{\frac{1}{2}}\right)^8 = 2^4 = 16$$

$$= \sqrt{2}\cdot 16 \left(\cos\left(-\frac{9}{4}\pi\right) + i \operatorname{sen}\left(-\frac{9}{4}\pi\right)\right)$$

$$= \sqrt{2}\cdot 16 \left(\cos\left(\frac{7}{4}\pi\right) + i \sin\left(\frac{7}{4}\pi\right)\right)$$

$$= \sqrt{2}\cdot 16 \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)$$

$$= 16 - 16i$$

$$\cancel{\sqrt{2}}\cdot 16\cdot \frac{1}{\cancel{\sqrt{2}}} - \cancel{\sqrt{2}}\cdot 16\cdot \frac{i}{\cancel{\sqrt{2}}}$$

Nota

$$z^n = |z|^n e^{in\theta} = |z|^n \left(\cos(n\theta) + i \sin n\theta\right)$$

Radici n-sime

$$\sqrt[n]{|z|}\, e^{i\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)} = \sqrt[n]{|z|}\left(\cos\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right) + i \sin\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)\right)$$

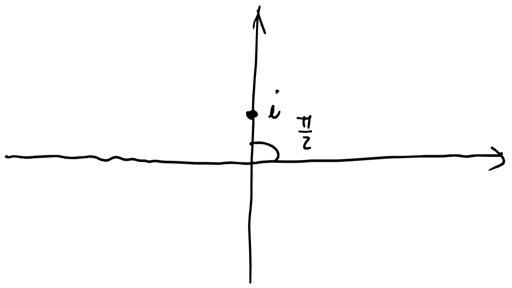
ESERCIZIO

$$z = \frac{2i-3}{2+3i}$$

1) Scrivere la forma algebrica di z.
2) Calcolare le radici cubiche di z.

1) $z = i$

2) $x = 0 \quad y = 1$



$|z| = \sqrt{x^2+y^2} = \sqrt{1} = 1$

$\varphi = ?$

$\cos\varphi = \frac{0}{1} = 0$ e $\operatorname{sen}\varphi = \frac{1}{1} = 1 \Rightarrow \varphi = \frac{\pi}{2}$

Radici cubiche $(n=3)$

$\varphi = \frac{\pi}{2}$

$\frac{\varphi}{3} = \frac{\frac{\pi}{2}}{3} = \frac{\pi}{2}\cdot\frac{1}{3} = \frac{\pi}{6}$

$\sqrt[3]{|z|}\left(\cos\left(\frac{\varphi}{3} + \frac{2k\pi}{3}\right) + i\operatorname{sen}\left(\frac{\varphi}{3} + \frac{2k\pi}{3}\right)\right)$

$= \sqrt[3]{1}\left(\cos\left(\frac{\pi}{6} + \frac{2k\pi}{3}\right) + i\operatorname{sen}\left(\frac{\pi}{6} + \frac{2k\pi}{3}\right)\right)$

$= \cos\left(\frac{\pi}{6} + \underbrace{\frac{2k\pi}{3}}\right) + i\operatorname{sen}\left(\frac{\pi}{6} + \frac{2k\pi}{3}\right). \qquad k = 0, 1, 2.$

- $k = 0$

  $\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} + i\,\frac{1}{2}$

$\frac{\pi}{6} + \frac{2k\pi}{3} = \frac{\pi + 4k\pi}{6}$ → $\frac{\pi}{6}$ $(k=0)$; $\frac{5}{6}\pi$ $(k=1)$; $\frac{9}{6}\pi = \frac{3}{2}\pi$ $(k=2)$

- $k = 1$

  $\cos\left(\frac{5}{6}\pi\right) + i\sin\left(\frac{5}{6}\pi\right)$

  $= -\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}$

- $k = 2$

  $\cos\left(\frac{3}{2}\pi\right) + i\sin\left(\frac{3}{2}\pi\right) = 0 + i\cdot(-1) = -i$

ESERCIZIO

- Si consideri il numero $z = \frac{3i - 1}{1+i}$
  1) Scrivere la forma algebrica di z.
  2) Scrivere la forma algebrica delle radici cubiche di $6i - 1 + z$

---

1) $z = \frac{3i-1}{1+i} = \frac{3i-1}{1+i}\cdot\frac{1-i}{1-i} = \frac{3i - 1 + 3 + i}{2} = \frac{4i+2}{2} = 1 + 2i$

2) Sia $w = 6i - 1 + z$

$= 6i \cancel{-1} + \cancel{1} + 2i = 8i$

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$$= 6i \cancel{-1} + \cancel{1} + 2i = 8i$$

$x = 0 \quad , \quad y = 8$

$$|z| = \sqrt{x^2+y^2} = \sqrt{8^2} = 8$$

$$\varphi = \frac{\pi}{2} \qquad \left(\cos\varphi = \frac{0}{8} = 0, \quad \sin\varphi = \frac{8}{8} = 1 \right)$$

Radici cubiche:

$$\sqrt[3]{|w|} \left(\cos\left(\frac{\varphi}{3} + \frac{2k\pi}{3} \right) + i \sin\left(\frac{\varphi}{3} + \frac{2k\pi}{3} \right) \right)$$

$$= \sqrt[3]{8} \left(\cos\left(\frac{\pi}{6} + \frac{2k\pi}{3} \right) + i \operatorname{sen}\left(\frac{\pi}{6} + \frac{2k\pi}{3} \right) \right) \qquad k = 0, 1, 2.$$

$$\frac{\pi}{6} + \frac{2k\pi}{3} = \begin{cases} \frac{\pi}{6} & (k=0) \\ \frac{5}{6}\pi & (k=1) \\ \frac{3}{2}\pi & (k=2) \end{cases}$$

Radici cubiche:

$k = 0$

$$2 \left(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6} \right) = 2 \left(\frac{\sqrt{3}}{2} + i \frac{1}{2} \right) = \sqrt{3} + i$$

$k = 1$

$$2 \left(\cos\frac{5\pi}{6} + i \sin\frac{5}{6}\pi \right) = 2 \left(-\frac{\sqrt{3}}{2} + i \frac{1}{2} \right) = -\sqrt{3} + i$$

$k = 2$

$$2 \left(\cos\left(\frac{3}{2}\pi\right) + i \sin\left(\frac{3}{2}\pi\right) \right) = 2 \left(0 + i \cdot (-1) \right) = -2i.$$